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Preface

This book grew from a course I gave at the Euler Institute for Discrete Mathematics and its Applications in the week 10–14 November 1997. The audience were keen to have notes, and so handwritten notes were produced each day. I have corrected and expanded these notes into the present version. The comments, questions and perplexities of the students on the course have been of very great value to me. In particular, they wanted to see proofs of theorems wherever possible, so I have included more proofs than I did in the lectures. Where a proof is too long to explain completely, I have given a sketch which tries to convey the main ideas, or worked a special case. Five chapters correspond to the five days of the course: introduction, character theory, the O'Nan–Scott Theorem, oligomorphic groups, and miscellanea.

Since the course, I have added a chapter on coherent configurations, two sections on computations with permutation groups, and tables of the finite simple groups and the finite 2-transitive groups.

I assume some knowledge of group theory and some mathematical sophistication. When other areas of mathematics (such as probability or logic) are invoked, I have tried not to assume any detailed knowledge: some results can be taken on trust.

The choice of topics is a bit idiosyncratic; this is not a complete treatment of the subject. Other topics which might have been chosen, and further details of those which do appear, are given in the references. For about thirty years, the only general reference on permutation groups was Helmut Wielandt's influential book [186]. In addition, Wielandt wrote several sets of lecture notes and many important papers; these are conveniently available now in Volume 1 of his *Mathematische Werke* [189]. In the last few years, more sources have become available, and I recommend the following as general references: the books by Bhattacharjee, Macpherson, Möller and

Neumann [17], Cameron [34], and Dixon and Mortimer [64], and the chapter on Permutation Groups in the Handbook of Combinatorics [36]. Passman's book [146] discusses such topics as the detailed structure of Frobenius and Zassenhaus groups.

I am grateful to EIDMA (especially Henny Houben and Henk van Tilborg) for the opportunity to give the course; to the students, especially Jürgen Müller and Max Neunhöffer, whose construction of the random graph is included in Chapter 5; to Colva Roney-Dougal, for working through the text and exercises, spotting many misprints and inclarities; to Leonard Soicher, for help with the worked examples using GAP; to Sasha Ivanov and Joachim Neubüser, for the history of cellular algebras and of classification of permutation groups; and to two anonymous referees, for helpful comments.

I have provided many exercises. Some of them are not straightforward; in cases where these are published results, references to the literature have been given. Hints are usually included.

In several cases, I have referred to sources of information available on the Internet. There is also a World Wide Web page associated with the book, at the URL

`http://www.maths.qmw.ac.uk/~pjc/permgps/`

This will contain the links mentioned in the book (and others), and possibly solutions to the exercises.

The GAP programming examples in Chapters 1 and 3 have been tested with the current release (version 3.4.4) of GAP. They also work with the beta4 version of GAP 4 (using an unofficial version of `GRAPE`). It is hoped that, by the time this book is published, GAP 4 will be officially available and the examples will work correctly. News of this will be posted on the Web page.

Peter J. Cameron
9 October 1998