Fifty years around Jan: Some history and some mathematics

Peter J. Cameron, University of St Andrews

Saxl Day
Cambridge, 22 July 2022

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The same event brought Jan to Britain. He learned of the invasion while on a train returning home after a holiday here; he got off the train and returned to Britain.
Both of us made our homes here.

The 1970s

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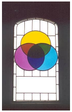
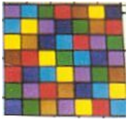
Fellows of Caius, including John Venn, Ronald Fisher and John Conway, made appearances in the lectures.





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But I am honoured to be a member of a college that had Jan as a fellow (as well as the other distinguished people I mentioned).

Our joint papers

I wrote four joint papers with Jan:

- P. J. Cameron, P. M. Neumann and J. Saxl, An interchange property in finite permutation groups, *Bull. London Math. Soc.* **11** (1979), 161–169.
- P. J. Cameron and J. Saxl, Permuting unordered subsets, *Quart. J. Math. Oxford* 2 **34** (1983), 167–170.
- P. J. Cameron, P. M. Neumann and J. Saxl, On groups with no regular orbits on the set of subsets, *Arch. Math.* **43** (1984), 295–296.
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The fourth is of course the most celebrated of the four.

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The soluble conjugacy class graph of a finite group is the graph whose vertices are the conjugacy classes, two vertices C and D joined if there exist $g \in C$ and $h \in D$ such that $\langle g, h \rangle$ is a soluble group.

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For Landau's theorem, there are now explicit bounds. Such bounds for our theorem are not known: an open problem for somebody to tackle.

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Work is still in progress, but here are a few of our results.

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- *Pre-primitivity is closed upwards.*
- *A regular permutation group G is pre-primitive if and only if it is a Dedekind group, that is, all subgroups are normal.*
- *A wreath product of transitive groups (in its imprimitive action) is pre-primitive if and only if the two factors are pre-primitive.*

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Perhaps there are other hierarchies of permutation group properties where similar ideas can be applied …

Farewell Jan