Simon Norton, Norton algebras and spherical designs

Peter J. Cameron

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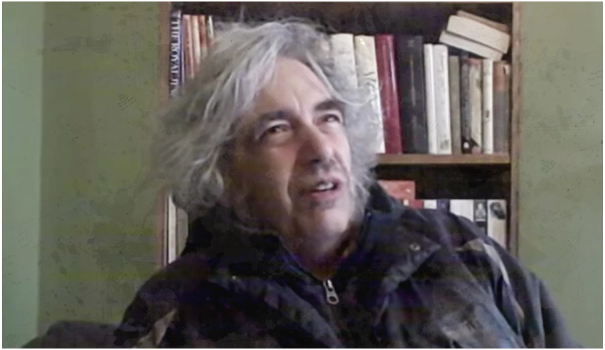
$$21493760 = 21296876 + 196883 + 1$$

$$864299970 = 842609326 + 21296876 + 196883 + 196883 + 1 + 1$$

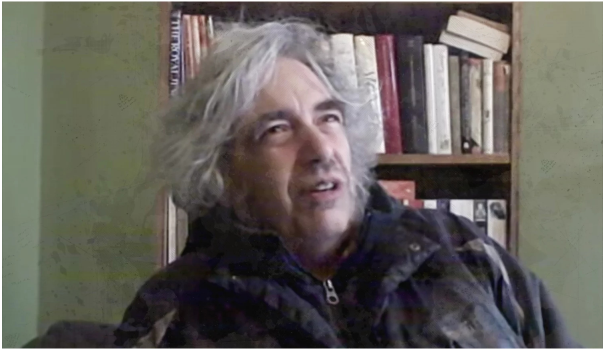
Simon Norton lecture, London, 12/02/2024

Also in memory of Richard Parker, who died last month

Simon P. Norton, 1952–2019



Simon P. Norton, 1952–2019



Simon Norton was a remarkable person. From early success in the Mathematical Olympiad, he made his mark on mathematics, in both group theory and game theory, and also contributed to the Campaign for Better Transport.

His most memorable mathematical work was done in close association with John Conway. It is no surprise that both their biographers use the word “genius” to describe their subjects:

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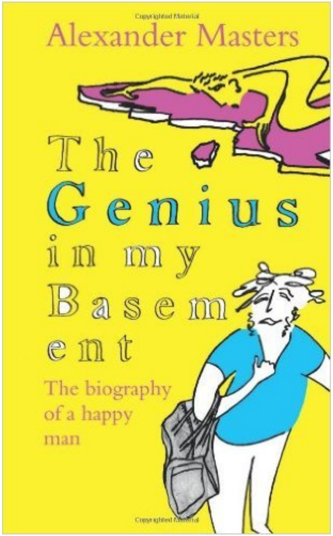
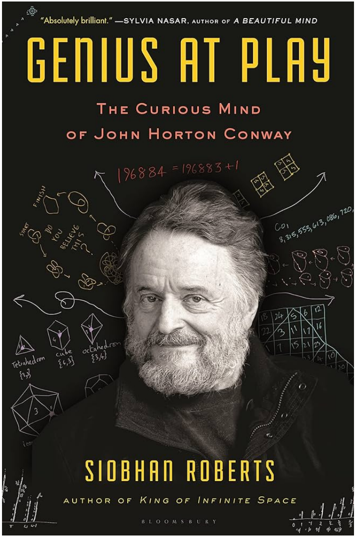
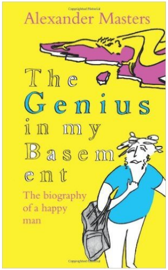
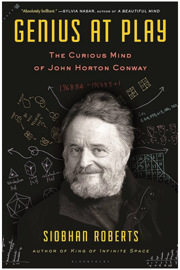




Conway and Norton

But look more closely at the covers:





Conway was playful, indeed some felt that he did little but play. I am not sure whether Norton ever played, but he took very great pleasure in what he did; I think Masters is right to describe him as a happy man.

Groups

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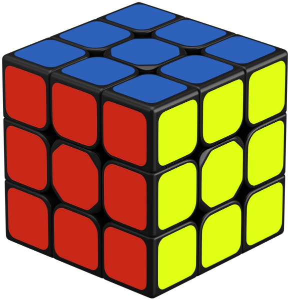
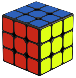
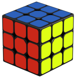
Here is an object which may be familiar, the Rubik cube:





A move consists of turning one of the six “faces” (made up of nine small cubes) through a quarter, half, or three-quarter turn. After a few moves the colours will seem thoroughly mixed up; indeed, the number of different configurations is 218963266577104896000. (We call this large number the order of the group associated with the cube.)



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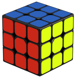
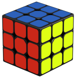
This is a big number! Yet there are people who can “solve” the cube (return it to its original configuration) in a few seconds. This is testimony to their insight into the structure of the group, but also to the efficiency of the algorithms for handling groups, and to a special feature of this group, which I now discuss.



When I was first given a Rubik cube, I devised a method of solving it, not very fast but it worked:

- first move the corner cubes to their correct positions (the centres of the faces don't move, and give us a reference frame: thus, with the face centres as shown, we know that the cube near us must be red-yellow-blue).



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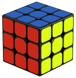
- then rotate the cubes if they are not in the right orientation;



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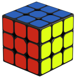

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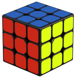

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At each stage, we have to do the task without spoiling the work which has already been done.

Simple groups

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In the 1960s, we knew several infinite families of simple groups, but there were just five "oddities" called sporadic groups which don't fit into any infinite pattern. Then all of a sudden, new sporadic groups started popping up all over the place. The largest of these was the Monster, a group of order 808017424794512875886459904961710757005754368000000000.

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This is *much* larger than the order of the Rubik cube group, but the difficulty of understanding it is compounded by the fact that it cannot be broken down into smaller groups, but has to be tackled head-on.

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Indeed, Conway and Norton played an important role in the story.

The ATLAS of Finite Groups

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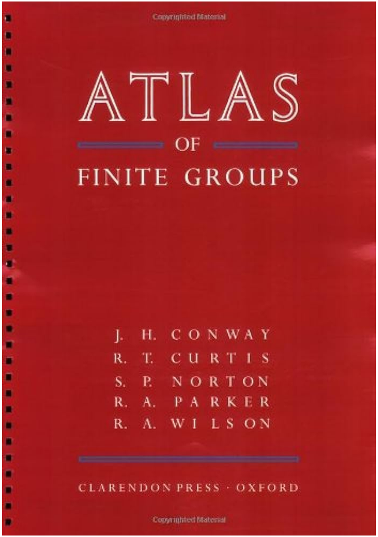
The (apocryphal) rules for authorship are well known: You must have two initials; your surname must have six letters with vowels in positions 2 and 5.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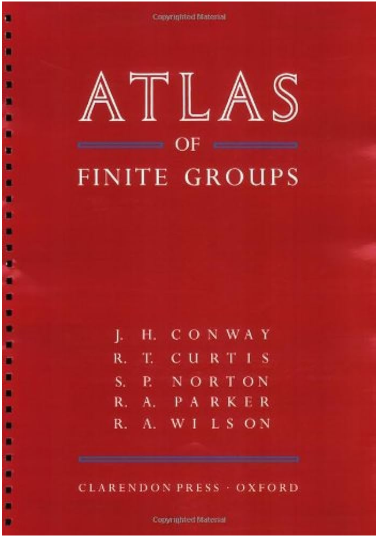

Simon left us just 30 publications. The ATLAS of Finite Groups has been cited 3857 times, according to Semantic Scholar.

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Part of the moonshine conjectures asserted that the three equations on my cover slide, namely

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were just the first of an infinite sequence of equations.
What is remarkable about these equations is that the left-hand sides are the Fourier coefficients of the classical modular function or *j*-function, whereas the right-hand sides are linear combinations of irreducible character degrees of the Monster.

The two sides of the Moonshine equations come from completely different branches of mathematics: the first is classic 19th century stuff (found in old analysis books); the second has been described as “21st century mathematics which fell into our laps in the 20th” (the numbers on the right are in the $\mathbb{ATLAS}$).

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The initial observation is due to John McKay, one of the few people who knew enough about both topics to recognise this.

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The paper contained things which are still unexplained. For example, the constant term of the j-function is 744, which doesn't fit the Moonshine pattern; but Conway and Norton point out that it is three times the dimension of the largest exceptional simple Lie algebra E_8. (Another remarkable coincidence, whose significance still eludes us.)

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The information for the second of these is summarised in the character table of the group; in the case of the Monster, this is a square array of size 194×194, *much* smaller than its multiplication table! This table was computed in the late 1970s by Fischer, Livingstone and Thorne, and Norton knew it very well indeed. The first column contains the numbers on the right in the Moonshine equations.

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The usual approach to understanding the subgroups of a group involve knowing the maximal subgroups and working down. But in the Monster there are certain small subgroups from which we can build up. Norton's papers on the anatomy of the Monster showed deep and detailed knowledge of these.

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This, and the departure of John Conway to Princeton in 1986, marked the end of the high point of Norton’s creativity. He continued to investigate the Monster, but spent more time on other interests, in particular public transport. (He was a strong supporter of this, contributing to campaigns to save bus routes, and making elaborately planned trips around Britain based on his deep knowledge of local bus timetables.)

A digression

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Mathematically, a football consists of 60 points marked on the surface of a sphere, with the lines joining them forming 12 pentagons and 20 hexagons. This is an example of a polyhedron (not one of the five regular polyhedra found by the ancient Greeks, but an important one anyway).

Your friendly neighbourhood chemist will probably tell you that it is a molecule of a form of carbon, with chemical formula C_{60}, which they call buckminsterfullerene, since it resemebles one of Buckminster Fuller's geodesic domes.

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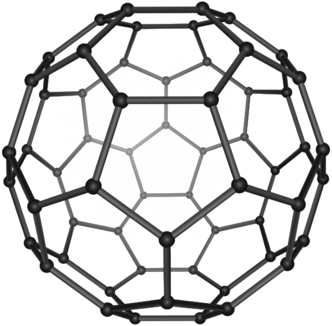

Roundness

The point about a football is that it is round, so that it rolls well. Imagine, if you can, playing football with a ball that had a lead weight attached at some point inside, or with one of the pentagons very much bigger than all the others. It would be quite a different game!

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A measure of "roundness" for a finite set of points on a sphere was devised by Philippe Delsarte, Jean-Marie Goethals and Jaap Seidel. They defined the notion of a spherical t-design, where t is a positive integer; the larger t, the more "round" the set of points is.

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To begin, imagine that we put a unit mass at each point of the set. There will be a centre of mass, which is found by averaging the coordinates of the points. We would like this to be at the origin, otherwise the set will be eccentric (like a bowling ball), and not suitable as a football.

Another way of saying this is that any linear function sums to zero over the points of the set. So the average of a linear function over a finite set is equal to its average over the whole sphere.

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Delsarte, Goethals and Seidel say that a finite set on the sphere is a spherical t-design if, for any polynomial function of degree at most t, its average over the point set is equal to its average over the whole sphere. So the two conditions above define spherical 1- and 2-designs.

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This is useful in numerical integration. The integral of a function over the sphere can be approximated by its sum over a finite point set (suitably normalised); if the set is a spherical t-design then the approximation gives the exact value for polynomial functions of degree at most t.

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This experiment didn't require a multi-billion dollar particle accelerator, but was done with tabletop equipment in the laboratory!

A better football?

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The calculations involve mathematics from the nineteenth century, namely invariant theory of the icosahedral group. The vertices of the "improved" football are zeros of the invariant of degree 6 for this group.

MATHEMATICS

P. J. CAMERON, J.-M. GOETHALS AND J. J. SEIDEL

The Krein condition, spherical designs, Norton algebras and permutation groups

Dedicated to N. G. de Bruijn on the occasion of his sixtieth birthday

Communicated by Prof. J. H. van Lint at the meeting of October 29, 1977

Merton College, Oxford, England
M.B.L.E. Research Laboratory, Brussels, Belgium
Technological University, Eindhoven, The Netherlands

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Our reference for the attribution was “personal communication from J. H. Conway”.

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I cannot explain this in detail but will give a rough idea with minimum technical detail.

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Now consider the action of G on $V \otimes V$. This can be decomposed as a sum of irreducible representations. We ask,

> *Does $V \otimes V$ have an irreducible constituent which is isomorphic to V?*

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If it is "yes", then there is a G-invariant map from $V \otimes V$ to V. This is a binary operation of "multiplication" or "composition" on V, and this is what we called a Norton algebra.
I will try to give a handwaving explanation. A perfectly smooth sphere offers no handholds. But if the object departs from roundness in some way, we can perhaps get a grip on it and come to an understanding of it. This was essentially what happened.

The Griess algebra

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This algebra has been crucial to further study of the Monster, for example in the proof of the Moonshine conjectures by Richard Borcherds, and the work of Sasha Ivanov and colleagues on Majorana algebras.

But that is the end of my story.

… for your attention.